Comparison between Fast Steady-State Analysis Methods for Time-Periodic Nonlinear Magnetic Field Problems

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Abstract — This paper investigates the effectiveness of various methods to accelerate the convergence to a steady state in time-periodic electromagnetic field problems. We introduce the time-periodic explicit error correction (TP-EEC) method considering both dc and linearly-varying error components. The advantages and disadvantages of the TP-EEC method and the time differential correction (TDC) method are discussed from the standpoints of computational costs and the application scope.

I. INTRODUCTION

In transient analyses of time-periodic electromagnetic field problems, several numerical methods to attain the steady state solutions efficiently have been proposed. For instance, the time-periodic finite-element method (TPFEM) [1][2] and the harmonic balance method [3] can obtain steady state solutions directly. However, it requires huge computational costs because of the large coefficient matrix. On the other hand, although the time-periodic explicit error correction (TP-EEC) method [4] can accelerate the convergence to a steady state drastically, their performance has not been discussed deeply from the theoretical standpoints. In addition, the shooting method [5], which was originally developed for the electronic circuit analysis, appears similar to the TP-EEC method in the basis of error correction. Therefore, it is indispensable to investigate the advantages and disadvantages of the above methods and clarify their theoretical relationship.

In this paper, we first introduce the TP-EEC method taking account of both the dc and linearly-varying error components. Then, we compare the TP-EEC methods with the time differential correction (TDC) method [6], which extracts fundamental harmonic components of steady state solutions by making use of the differential of transient solutions, from the standpoints of the computational costs and the scope of application. Numerical results that clarify the features of the above methods are also presented.

II. FAST STEADY-STATE ANALYSIS METHODS

A. TP-EEC Method

A nonlinear system of equations in the A-ϕ formulation in a quasi-static field is given by

$$S(\boldsymbol{x})+C\frac{\partial}{\partial t}\boldsymbol{x}=\boldsymbol{f}\,, \tag{1}$$

where $\boldsymbol{x}$ is the unknown vector, and $\boldsymbol{f}$ is the right-hand-side vector. The number of unknowns is m. $S(\boldsymbol{x})$ is generally nonlinear with respect to $\boldsymbol{x}$ because of nonlinear magnetic properties and C is constant. One or half period is divided into n time steps. In this paper, we treat electromagnetic field problems which satisfy $\boldsymbol{x}_i = \pm\boldsymbol{x}_{i+n}$ and $\boldsymbol{f}_i = \pm\boldsymbol{f}_{i+n}$ in the steady state. Here, the subscripts indicate the time step, and the upper and lower signs of $\pm$ correspond to the ordinary and half time-periodic conditions.

The key to the TP-EEC method is choosing the auxiliary matrix B [7] to successfully extract poorly converged error components which correspond to the large time constants of an analyzed system. When we consider dc components as poorly-converged error, B which can extract dc error components is given by

$$B=\begin{bmatrix} I & \cdots & I & \cdots & I\end{bmatrix}^T, \tag{2}$$

where I is an m by m identity matrix and B is an nm by m matrix. By applying the SD-EEC method [7]-[9] to the TPFEM with B in (2) and using the θ method for a time integration scheme, we obtain a linear system of auxiliary equations as follows [4]:

$$\left[\sum_{i=1}^{n} S_i+(1\mp 1)\widetilde{C}_n\right]\boldsymbol{p}_0=-\widetilde{C}(\boldsymbol{x}_0)\pm\widetilde{C}(\boldsymbol{x}_n)=\boldsymbol{r}_0\,, \tag{3}$$

$$\widetilde{T}_i=\theta S_i+\frac{C}{\Delta t},\ \widetilde{C}_i=-(1-\theta)S_i+\frac{C}{\Delta t},\ S_i=\frac{\partial S}{\partial \boldsymbol{x}}(\boldsymbol{x}_i)\,, \tag{4}$$

$$\widetilde{C}(\boldsymbol{x}_i)=-(1-\theta)S(\boldsymbol{x}_i)+\frac{C}{\Delta t}\boldsymbol{x}_i\,,\ \widetilde{T}_i-\widetilde{C}_i=S_i \tag{5}$$

where $\boldsymbol{p}_0$ indicates the extracted dc error component. The upper and lower signs of $\mp$ and $\pm$ in (3) correspond to the ordinary and half time-periodic problems, respectively.

When we take into account both dc and linearly-varying error components, B can be written as

$$B=\begin{bmatrix} I & \cdots & I & \cdots & I \\ \left(-1+\dfrac{2}{n}\right)I & \cdots & h_i I & \cdots & I\end{bmatrix},\ h_i=\frac{2i-n}{n}\quad (1\le i\le n)\,. \tag{6}$$

Based on B in (6), the auxiliary system which can extract both dc and linearly-varying error components is given by

$$\begin{bmatrix} \left[\sum_{i=1}^{n} S_i+(1\mp 1)\widetilde{C}_n\right] & \left[\sum_{i=1}^{n} h_i S_i+(1\mp 1)\widetilde{C}_n\right] \\ \left[\begin{aligned}&\sum_{i=1}^{n}\left(h_i S_i-\frac{2}{n}\widetilde{C}_i\right)\\&+\left(1\mp h_1+\frac{2}{n}\right)\widetilde{C}_n\end{aligned}\right] & \left[\begin{aligned}&\sum_{i=1}^{n}\left(h_i^2 S_i-\frac{2h_i}{n}\widetilde{C}_i\right)\\&+\left(1\mp h_1+\frac{2}{n}\right)\widetilde{C}_n\end{aligned}\right]\end{bmatrix}\begin{pmatrix}\boldsymbol{p}_0\\ \boldsymbol{p}_1\end{pmatrix}=\begin{pmatrix}\boldsymbol{r}_0\\ h_1\boldsymbol{r}_0\end{pmatrix}, \tag{7}$$

where $\boldsymbol{p}_1$ indicates the linearly-changed error component. Although the error correction based on (7) has better performance than that based on (3), the coefficient matrix in (7) is nonsymmetric and therefore the double memory usage and a nonsymmetric iterative solver are required.

The procedure of the TP-EEC method is as follows:

(i) Perform a transient analysis starting with $\boldsymbol{x}_0$ for one or a half period.

(ii) Obtain the correction vectors by solving (3) or (7).

(iii) Update $\boldsymbol{x}_n$ by $\boldsymbol{x}_n \leftarrow \boldsymbol{x}_n + \boldsymbol{p}_0$ or $\boldsymbol{x}_n \leftarrow \boldsymbol{x}_n + \boldsymbol{p}_0 + \boldsymbol{p}_1$ and substitute into $\boldsymbol{x}_0$.

(iv) If the iteration does not converge, go to (i).

B. Time Differential Correction Method

First, we can remove the large high-order harmonic components in the steady-state solutions by integrating transient solutions as follows:

$$\langle \boldsymbol{x}_1 \rangle = \frac{1}{2\phi}\int_{\theta-2\phi}^{\theta} \boldsymbol{x}(\theta')\,d\theta' = \frac{\sin\phi}{\phi}\boldsymbol{x}(\theta-\phi), \qquad (8)$$

where $\theta=\omega t$ and 2ϕ is the integral interval corresponding to the period of the high-order harmonic component. Then, we can eliminate exponentially-decreased error components corresponding to large time constants and extract fundamental harmonic components of steady state solutions by differentiating $<\boldsymbol{x}_1>$ as

$$\boldsymbol{x}_{\text{new}}(\theta-\phi) = -\frac{\phi}{\sin\phi}\frac{d\langle \boldsymbol{x}_{\text{old}} \rangle}{d\theta^2}. \qquad (9)$$

When the steady state solutions do not include high-order harmonic components, averaging procedure using (8) is unnecessary. The TDC method has the advantage of low computational cost compared with the TP-EEC method which needs solving linear system of equations to extract error components. However, the TDC method is not suitable for the problems which include many high-order harmonic components in the steady state solutions such as rectangular wave form because it focuses on only the fundamental harmonic component.

III. NUMERICAL EXAMPLES

Fig. 1 shows the analyzed model. The number of elements and unknowns are 32,000 and 100,880, respectively. One period is divided into 40 time steps. All the computations were performed on Intel Core i7-975 Extreme Edition/3.33GHz, 12.0 GB RAM. The relative errors of eddy-current loss W_e with respect to those obtained from the TPFEM is shown in Fig. 2. Table I shows the number of time steps and the calculation time required for obtaining steady state solutions for a half period. We judge the steady state solutions are obtained when all the relative errors of W_e in a half period are less than 1 %. Because this problem does not include large high-order harmonic components in the steady state solutions, the best performance is obtained from the TDC method. Although the TP-EEC method based on (7) needs large computational costs compared with that based on (3), it has better performance in convergence acceleration.

The error analysis of the TP-EEC methods based on (3) and (7), the relationship between the TP-EEC method and the shooting method, and the advantage and disadvantage of the various fast steady-state analysis methods will be discussed in the full paper.

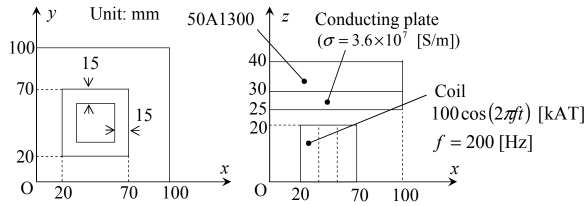

Fig. 1. Analyzed model (1/8 part of the whole model).

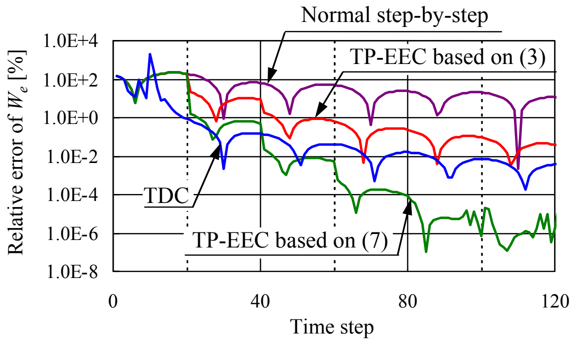

Fig. 2. Relative error of eddy-current loss.

TABLE I
COMPARISON OF FAST STEADY-STATE ANALYSIS METHODS

	TP-EEC based on (3)	TP-EEC based on (7)	TDC
Time steps	63	44	38
CPU time [s]	170.3	140.2	108.2

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